The Triginometric Functions of a real number establish a system for computation. These functions have properties that can simplify computations under some conditions, but other functions cannot be simplified. They are found naturally in the measuring of distance and angles as an example of practical trigonometry. In situations where the activity is periodic the practical domain of the function becomes the set of real numbers between $-\pi$ and π, inclusive. If t is a Trigonometric Function then $t(u) = t(u \pm 2\pi)$.
Let u be any real number. Consider u to be the size of an angle in radians. Define the *Trigonometric Functions* (of u) as: $\sin u$, $\cos u$, $\tan u$.

The following are *Trigonometric* (functional) *Indentities.*

Reciprocal: (multiplicative inverses exist by construction)

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{\sin u}{\cos u} = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u} = \frac{1}{\tan u}$$

Even-or-Odd Parity: (additive inverses exist by construction)

$$\sin(-u) = -\sin(u) \quad \cos(-u) = \cos(u) \quad \tan(-u) = -\tan(u)$$

$$\csc(-u) = -\csc(u) \quad \sec(-u) = \sec(u) \quad \cot(-u) = -\cot(u)$$

Co-Function: (some linear transformations exist by construction)

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad \cos\left(\frac{\pi}{2} - u\right) = \sin u \quad \tan\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u \quad \sec\left(\frac{\pi}{2} - u\right) = \csc u \quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

Pythagorean: (there are metric properties by construction)

$$\begin{aligned}
\sin^2 u + \cos^2 u &= 1 \\
\csc^2 u - \cot^2 u &= 1 \\
\sec^2 u - \tan^2 u &= 1
\end{aligned}$$

Double-Angle: (scalar multiplication by 2 exists by construction)

$$\begin{aligned}
\sin(2u) &= 2\cos(u)\sin(u) \\
\cos(2u) &= \cos^2(u) - \sin^2(u) \\
&= 2\cos^2(u) - 1 \\
&= 1 - 2\sin^2(u) \\
\tan(2u) &= 2\left(\frac{\tan(u)}{1 - \tan^2(u)}\right)
\end{aligned}$$

Half-Angle: (some square roots of functions exist by construction)

$$\begin{aligned}
\sin^2(u) &= \frac{1 - \cos(2u)}{2} \\
\cos^2(u) &= \frac{1 + \cos(2u)}{2} \\
\tan^2(u) &= \frac{1 - \cos(2u)}{1 + \cos(2u)}
\end{aligned}$$

Sum-and-Difference: (functions of sums exist by construction)

$$\begin{aligned}
\sin(u+v) &= \sin(u)\cdot\cos(v)+\cos(u)\cdot\sin(v)\\
\sin(u-v) &= \sin(u)\cdot\cos(v)-\cos(u)\cdot\sin(v)
\end{aligned}$$

$$\begin{aligned}
\cos(u+v) &= \cos(u)\cdot\cos(v)-\sin(u)\cdot\sin(v)\\
\cos(u-v) &= \cos(u)\cdot\cos(v)+\sin(u)\cdot\sin(v)
\end{aligned}$$

$$\begin{aligned}
\tan(u+v) &= \frac{\tan(u)+\tan(v)}{1-\tan(u)\cdot\tan(v)}\\
\tan(u-v) &= \frac{\tan(u)-\tan(v)}{1+\tan(u)\cdot\tan(v)}
\end{aligned}$$

Sum-to-Product Conversion: (sums of functions exist by construction)

$$\begin{aligned}
\sin u+\sin v &= 2\left[\sin\left(\frac{u+v}{2}\right)\cdot\cos\left(\frac{u-v}{2}\right)\right]\\
\sin u-\sin v &= 2\left[\cos\left(\frac{u+v}{2}\right)\cdot\sin\left(\frac{u-v}{2}\right)\right]\\
\cos u+\cos v &= 2\left[\cos\left(\frac{u+v}{2}\right)\cdot\cos\left(\frac{u-v}{2}\right)\right]\\
\cos u-\cos v &= -2\left[\sin\left(\frac{u+v}{2}\right)\cdot\sin\left(\frac{u-v}{2}\right)\right]
\end{aligned}$$

Product-to-Sum Conversion: (products of functions exist by construction)

$$\begin{aligned}
\sin u\cdot\cos v &= \frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]\\
\cos u\cdot\sin v &= \frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]\\
\cos u\cdot\cos v &= \frac{1}{2}\left[\cos(u+v)+\cos(u-v)\right]\\
\sin u\cdot\sin v &= -\frac{1}{2}\left[\cos(u+v)-\cos(u-v)\right]
\end{aligned}$$