A Spoonful of Spinors

"No one fully understands spinors. Their algebra is formally understood but their general significance is mysterious. In some sense they describe the 'square root' of geometry and, just as understanding the square root of -1 took centuries, the same might be true of spinors."

—*Sir Michael Atiyah*

The context for this note is the three-dimensional space of classical physics. Incorporated are the real numbers used to measure, and compute, the time.

Let $\mathbf{v} = (x, y, z) \longleftrightarrow \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} = \mathbf{M}$. ($\mathbf{v}$ is a vector; $\mathbf{M}$ is a matrix.)

Define $\mathbf{v^2} = (x, y, z) \times (x, y, z) = (x^2, y^2, z^2)$

$$\longleftrightarrow \begin{bmatrix} x^2 & 0 & 0 \\ 0 & y^2 & 0 \\ 0 & 0 & z^2 \end{bmatrix} = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} = \mathbf{M}^2.$$

Define $|\mathbf{v}| = |(x, y, z)| = \sqrt{x^2 + y^2 + z^2}$ (conventional distance formula)

$$\longleftrightarrow \sqrt{\operatorname{trace}\left(\begin{bmatrix} x^2 & 0 & 0 \\ 0 & y^2 & 0 \\ 0 & 0 & z^2 \end{bmatrix} \right)} = \sqrt{\operatorname{tr}(\mathbf{M}^2)}\,.$$

And assume that $x \neq 0, y \neq 0, z \neq 0$ ($i.e., xyz \neq 0$).
The *trace* of a matrix is the sum of its entries on the diagonal.

If $\det(\mathbf{M}) = xyz = 0$ then suppose that $yz = 1$. Thus $x = 0$. In this case, we are left with a reduced domain of only y and z alone, as both are now separated from x ($i.e., y \neq 0, z \neq 0$).

Let E_1, E_2, E_3 denote the Euclidean spaces of 1,2,3–dimensions. Note that $E_1 \subset E_2 \subset E_3$. Suppose that $(y, z) \in E_2$. Then $(0, y, z)$ is a *spinor* in E_3: *i.e.,* $(0, y, z) \leftrightarrow yz = 1 = |e^{it}|$, $t \in E_1$. Further, $|e^{it}|$ is represented graphically by a unit circle in the *x,y*–plane. Spinors are a type of mathematical object in Complex numbers that can be embedded into a higher-dimensional Euclidean space.

Discussion.

The set product of two unit circles at right angles is the unit sphere. In a sense, these two circles are the 'square roots' of the geometry of the sphere. They look like the two orthogonal great circles that form the fours principal (*i.e.,* 0, 90, 180 and 270 degree) meridians of a globe. They do not include the equator, which depends on the axis of rotation.

These spinors are not numbers; they are sets of numbers. Associated with these sets of numbers is a *geometry* having distances and angles. This particular geometry is equivalent to, and very much like, classical analytic geometry; but it is not identical to it. It is a discrete geometry. It is the geometry of the Quantum Mechanics, and it is non-Euclidean.

The classical (*i.e.,* standard) analytic geometry is the foundation of the Relativistic Mechanics. Smoothly blending these geometries is the goal of the Grand Unification Theory. The algebra is the same for both the Quantum Mechanics and for the Relativistic Mechanics, but the details of the calculus change. Any calculus is deeply dependent on the set of points that it manipulates with its functions and their transformations.

The continuous infinity of the Relativistic Mechanics is that of the Real numbers. The countable infinity of the Quantum Mechanics is that of the Integers. Of particular importance is that subset of the Integers known as the Natural (or counting) numbers, and its subset of the Prime numbers. A uniquely enigmatic prime number is 137, which is equal to $(3!)(23)-1$.

$\frac{1}{137}$ is numerically close to the measured value of α, the *Fine Structure Constant.* This constant, which is purely numerical and has no physical units, accounts for the wave-length of the Hydrogen radiation seen in all directions of the Universe from the earth. The measurements of the value of α are performed using the Relativistic—*not the Quantum*—Mechanics.

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